Elements of the transfer matrix $I^{(i)}$ are given by the following expressions:

$$T_{11}^{i} = \frac{1}{M_1}\left(\lambda_1 S_1 R_3 ch(\lambda_1 h_i) - \lambda_3 ch(\lambda_3 h_i)\right) \quad \text{(A6)}$$

$$T_{21}^{i} = \epsilon_i \frac{R_3}{M_1}\left(ch(\lambda_1 h_i) - ch(\lambda_3 h_i)\right) \quad \text{(A7)}$$

$$T_{31}^{i} = \epsilon_i \frac{1}{M_1}\left(S_1 R_3 sh(\lambda_1 h_i) - sh(\lambda_3 h_i)\right) \quad \text{(A8)}$$

$$T_{41}^{i} = \epsilon_i \frac{1}{M_1}\left(R_3 G_1 sh(\lambda_1 h_i) - G_3 sh(\lambda_3 h_i)\right) \quad \text{(A9)}$$

$$T_{12}^{i} = \frac{S_1 \lambda_1 \lambda_3}{M_1 \epsilon_i}\left(-ch(\lambda_1 h_i) + ch(\lambda_3 h_i)\right) \quad \text{(A10)}$$

$$T_{22}^{i} = \frac{1}{M_1}\left(-\lambda_3 ch(\lambda_1 h_i) + R_3 S_1 \lambda_1 ch(\lambda_3 h_i)\right) \quad \text{(A11)}$$

$$T_{32}^{i} = \frac{S_1}{M_1}\left(-\lambda_3 sh(\lambda_1 h_i) + \lambda_1 sh(\lambda_3 h_i)\right) \quad \text{(A12)}$$

$$T_{42}^{i} = \frac{1}{M_1}\left(-\lambda_3 G_1 sh(\lambda_1 h_i) + G_3 S_1 \lambda_1 sh(\lambda_3 h_i)\right) \quad \text{(A13)}$$

$$T_{13}^{i} = \frac{1}{M_2 \epsilon_i}\left(\lambda_1 S_1 G_3 sh(\lambda_1 h_i) - \lambda_3 G_1 sh(\lambda_3 h_i)\right) \quad \text{(A14)}$$

$$T_{23}^{i} = \frac{1}{M_2}\left(G_3 sh(\lambda_1 h_i) - G_1 R_3 sh(\lambda_3 h_i)\right) \quad \text{(A15)}$$

$$T_{33}^{i} = \frac{1}{M_2}\left(S_1 G_3 ch(\lambda_1 h_i) - G_1 ch(\lambda_3 h_i)\right) \quad \text{(A16)}$$

$$T_{43}^{i} = \frac{G_1 G_3}{M_2}\left(ch(\lambda_1 h_i) - ch(\lambda_3 h_i)\right) \quad \text{(A17)}$$

$$T_{14}^{i} = \frac{S_1}{M_2 \epsilon_i}\left(-\lambda_1 sh(\lambda_1 h_i) + \lambda_3 ch(\lambda_3 h_i)\right) \quad \text{(A18)}$$

$$T_{24}^{i} = \frac{1}{M_2}\left(-sh(\lambda_1 h_i) + R_3 S_1 sh(\lambda_3 h_i)\right) \quad \text{(A19)}$$

$$T_{34}^{i} = \frac{S_1}{M_2}\left(-ch(\lambda_1 h_i) + ch(\lambda_3 h_i)\right) \quad \text{(A20)}$$

$$T_{44}^{i} = \frac{1}{M_2}\left(-G_1 ch(\lambda_1 h_i) + G_3 S_1 ch(\lambda_3 h_i)\right) \quad \text{(A21)}$$

where

$$M_1 = R_3 S_1 \lambda_1 - \lambda_3 \qquad M_2 = S_1 G_3 - G_1$$

$$G_1 = \frac{\kappa_i k_0 S_1 + \lambda_1}{\mu_i} \qquad G_3 = \frac{\kappa_i k_0 + R_3 \lambda_3}{\mu_i} \quad \text{(A22)}$$

$$h_i = x_i - x_{i-1}$$

whereas

$$S_1 = -\frac{1}{k_0 \kappa_i \lambda_1}\left[\lambda_1^2 + \mu_i\left(\delta + k_0^2 \epsilon_i\right)\right] \quad \text{(A23)}$$

$$R_3 = \frac{\mu_i}{k_0 \kappa_i \epsilon_i \lambda_3}\left(\lambda_3^2 + \delta + k_0^2 \epsilon_i \mu_{\text{eff}_i}\right) \quad \text{(A24)}$$

$$\lambda_{1(3)} = \left\{\frac{1}{2}\left[g_2 \mp \left(g_2^2 - 4 g_0\right)^{1/2}\right]\right\}^{1/2} \quad \text{(A25)}$$

with

$$g_2 = -\delta(1+\mu_i) - 2k_0^2 \epsilon_i \mu_i \quad \text{(A26)}$$

$$g_0 = \mu_i\left(\delta + k_0^2 \epsilon_i \mu_{\text{eff}_i}\right)\left(\delta + k_0^2 \epsilon_i\right). \quad \text{(A27)}$$

Acknowledgment

The authors wish to thank L. Hayes for her assistance during preparation of this paper.

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Effects of Gain Compression, Bias Conditions, and Temperature on the Flicker Phase Noise of an 8.5 GHz GaAs MESFET Amplifier

C. P. LUSHER AND W. N. HARDY

***Abstract*—We have measured the phase noise of an 8.5 GHz GaAs MESFET amplifier at temperatures from 1.7 K to 300 K for input powers from −30 dBm to well past the 1 dB gain compression point and for sideband frequencies from 0.1 Hz to 25 kHz. The observed flicker phase noise was independent of input power, even at levels producing 4 dB of gain compression, and also changed very little with bias conditions. The intrinsic phase noise at low temperatures (observed below 2.17 K, where an extrinsic effect due to the bubbling of the liquid helium coolant disappears) was slightly higher than that observed at room temperature. However, we saw no sign of the dramatic increase in flicker phase noise at low**

Manuscript received April 6, 1988; revised October 5, 1988. This work was supported by the National Research Council of Canada and by the Natural Sciences and Engineering Research Council of Canada.

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IEEE Log Number 8825394.

temperatures recently reported at 9.7 GHz. Based on our data, we predict that a cryogenic loop oscillator built from such an amplifier should have exceptionally good short-term frequency stability.

I. Introduction

Small-signal microwave amplifiers based on GaAs MESFET devices exhibit very low noise temperatures at room temperature, with further improvement on cooling to liquid helium temperatures (noise temperatures of below 30 K at 10 GHz are possible at a physical temperature of 4.2 K [1]). Cooled GaAs FET devices have applications in the field of very high stability frequency sources (especially for those sources which make use of the improved stability offered by the cryogenic environment), for example, as preamplifiers in cryogenic H maser receivers and as active elements in cooled loop oscillators based on dielectrically loaded superconducting cavities. In these applications it is important that the device exhibit low phase noise.

It is well known that GaAs FET devices suffer from a high level of baseband flicker noise. In some GaAs MESFET devices the dominant low-frequency noise source has been found to be generation–recombination noise from deep level traps in the depletion regions [2]. Baseband noise, although relatively unimportant in small-signal operation at high frequencies, may appear in the microwave spectrum under conditions of device nonlinearity (for example, when the device is used as the active element in an oscillator). The near-carrier phase noise spectrum of GaAs FET oscillators is found in many cases to obey a $1/f^3$ law [3], [4], caused by low frequency $1/f$ device noise, which is mixed with the carrier signal via the nonlinearity of the device itself. There is evidence in the literature to suggest that the main contribution to up-conversion comes from a nonlinear gate–source capacitance C_{gs} [3]–[5]. We note, however, that it is not essential to invoke such a model for up-conversion of baseband noise in order to explain the near-carrier phase noise in FET loop oscillators since phase noise in the active device contributes directly to the spectrum, as described in Section IV. Nonlinear effects just give an additional contribution.

Recently Mann *et al.* [6] observed the near-carrier phase noise in *X*-band GaAs MESFET amplifiers to increase markedly on cooling, possibly due to an increase in the nonlinearity causing up-conversion (the low-frequency $1/f$ noise of certain GaAs FET devices has actually been observed to drop on cooling [7]). Subsequent to this investigation, phase noise measurements were made on a lower frequency (1.42 GHz) GaAs MESFET amplifier by Hürlimann and Hardy [8]. Once again, an increase in near-carrier phase noise was observed on cooling the amplifier to 4.2 K, but in this experiment the noise was seen to drop on cooling below the superfluid transition temperature for ^{4}He, $T_\lambda = 2.17$ K, to a level below that observed at room temperature. In this case the main contribution to phase noise above T_λ was coming from the bubbling of the liquid helium in the bath. However, the devices of Mann *et al.* [6] showed an increase in phase noise on cooling even when operated in a vacuum, ruling out the bubbling mechanism in their case.

Given that we were planning to use *X*-band GaAs FET amplifiers in high-performance cryogenic loop oscillators, it was essential to investigate possible reasons for these quite different results. Accordingly, we measured the phase noise of an 8.5 GHz amplifier as a function of input power level and bias conditions, as well as temperature. It appears from our measurements that a cooled loop oscillator of exceptionally good short-term frequency stability could be built from such an amplifier.

II. Amplifier Parameters

Our two-stage amplifier[1] uses Mitsubishi MGF 1412 devices in both stages (the amplifier used by Hürlimann had an MGF 1412 in its first stage and an MGF 1402 in its second stage). A regulated power supply[2] was used, which kept the drain currents at preset values by controlling the gate voltages. We observed no change in the phase noise when a different power supply (without feedback) was used instead; therefore we conclude that the dc power supply did not contribute to the measured phase noise. We used two different biasing conditions: (1) $V_{DS} \sim 3$ V, $I_D \sim 10$ mA; (2) $V_{DS} \sim 1.6$ V, $I_D \sim 6$ mA. The phase noise was found to vary little with power dissipation at all temperatures. We present here only results taken with the lower dissipation. With this bias setting the amplifier (which had Trak 60A6071 isolators[3] on the input and output) had a 3 dB bandwidth of 1.9 GHz, and a room-temperature gain of 19 dB at 8.5 GHz, rising to 24 dB at 4.2 K (where the 1 dB gain compression point occurred at an input power level of -21 dBm).

III. Results of Phase Noise Measurements

Phase noise measurements were made at 8.5 GHz using a phase bridge setup. An 8.5 GHz carrier was split and fed through the two arms of the bridge, one of which contained the amplifier under test (in a cryostat, with a 10 dB attenuator at its input). Extra attenuation could be introduced into this arm, at the top of the cryostat, in order to vary the input power. The other arm contained a phase shifter. The signals from the arms were mixed and brought to quadrature, where the mixer output (at dc) is most sensitive to fluctuations in the phase shift introduced by the amplifier. After some amplification, the mixer output was fed to an HP 3582A fast Fourier transform spectrum analyzer covering the range of sideband frequencies, f, from 0.1 Hz to 25 kHz. The background in the phase comparator was measured, for a given input power level and bias conditions, by replacing the amplifier arm with the attenuation required to bring the power level at the RF port of the mixer to the same value as that with the amplifier connected. The power into the LO port (which connected to the phase shifter) was independent of the amplifier input power level and bias conditions. In order to obtain low enough background it was necessary to use an 8.5 GHz source that had good amplitude as well as good frequency stability. In particular, the amplitude stability of the output from the $\times 4$ multiplier chain following our HP 8663A synthesizer (0.1 to 2.6 GHz) was not adequate directly: we used the output, which had good frequency stability, to phase-lock a YIG tuned oscillator. For sideband frequencies greater than 10 Hz, the background was always at least 5 dB below the level of the amplifier noise. The same was true below 10 Hz except in the case of the 300 K data taken with -30 dBm input power, where the amplifier noise was slightly below the level of the background at frequencies below 1 Hz. The background has been subtracted from all data presented in this paper.

The carrier power at the input to the amplifier (after the 10 db attenuator) was varied from -30 dBm to a level well above the 1 dB gain compression point (an upper limit of -10 dBm was used at room temperature, and of -16 dBm at all other temperatures). With these high input powers, 4 dB of gain compression

[1] Berkshire Technologies Inc., Oakland, CA, model X-8 5-20.
[2] Berkshire Technologies, Inc., model PS-3B.
[3] The contribution to the measured phase noise coming from the isolators was found to be negligible.

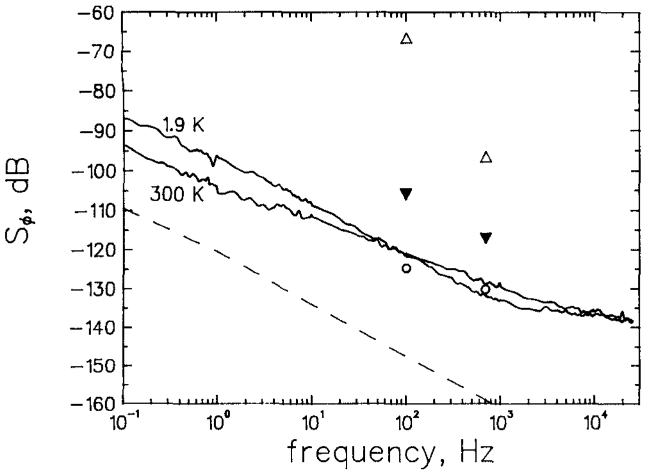

Fig. 1. Spectral density of phase fluctuations (relative to 1 rad^2/Hz) of an 8.5 GHz GaAs MESFET amplifier for different physical temperatures. Also shown are data from Mann *et al* [6] at input powers between -40 and -30 dBm: ○ 300 K; ▼ 4.2 K; △ 2 K. The dashed line is the 2 K data from [8].

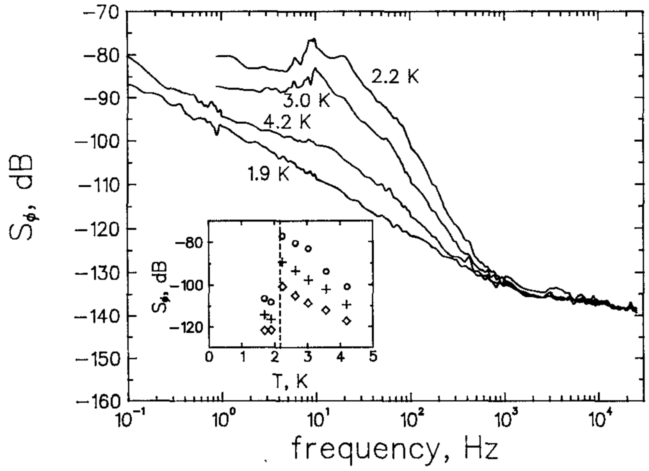

Fig. 2. Spectral density of phase fluctuations (relative to 1 rad^2/Hz) of an 8.5 GHz GaAs MESFET amplifier at temperatures between 1.7 K and 4.2 K. Inset shows the temperature dependence of S_φ at three different sideband frequencies: ○ 10 Hz; + 40 Hz; ◇ 100 Hz. Dashed line shows T_λ, the superfluid transition temperature of the liquid helium coolant.

was observed at 300 K and 4.2 K with the low bias settings. Within the range of input powers used, the observed noise power at the output was proportional to the power in the carrier at all temperatures. This corresponds to an additive phase noise independent of input power (we present here only our data taken with an input power of -30 dBm). This too was the result of Mann *et al.* [6] at room temperature and 77 K, and at these temperatures our data are in good agreement with theirs. In Figs. 1 and 2 we show the spectral density of phase fluctuations $S_\varphi(f)$ [9] for different physical temperatures. At temperatures of 4.2 K and below, the level of phase noise measured by Mann *et al.* [6] (taken at 9.7 GHz) was much higher than ours (50 dB higher at 100 Hz from the carrier for a -40 dBm input power and a temperature of 2 K, with an even larger discrepancy at higher input powers). On the other hand, the 2 K data of Hürlimann and Hardy [8] for a 1.42 GHz amplifier (the dashed line in Fig. 1) were around 25 dB lower than our data. The measured S_φ is presumably caused by fluctuations in the transit time of the signal through the device. Therefore, given the same spectral density of transit time fluctuations, one would expect S_φ at a given sideband frequency to be $\sim$15.5 dB lower at 1.42 GHz (and $\sim$ 2 dB higher at 9.7 GHz) than at 8.5 GHz. The remaining 10 dB of phase noise we observed may be due to an extra mechanism operating at our higher frequency or possibly just due to slightly different device characteristics. Also, the MGF 1402, used in Hürlimann's second stage, may have lower phase noise than the 1412 and this would further reduce the difference between the data measured here and those in [8].

Our data at 4.2 K and below were taken with the amplifier immersed in liquid helium. The effect of the boiling of the liquid on the phase noise measured above T_λ is clearly seen in Fig. 2, which shows $S_\varphi(f)$ at 4.2 K and below. On pumping the helium bath below 4.2 K the bubbling initially increases, becomes a maximum just above T_λ, and finally disappears completely below T_λ. S_φ at 10 Hz from the carrier drops by more than 30 dB on cooling through T_λ. The bubbles cause a random dielectric perturbation of the circuit impedances and thereby contribute to the measured phase noise. We note that in low phase noise applications this mechanism must be prevented (by operating the amplifier in a vacuum, for example).

The phase noise measured at 2 K is intrinsic to the devices (and not caused by the bubbling of the helium). It is actually somewhat higher than that observed at room temperature, in contrast to the data of Hürlimann and Hardy [8] at 1.42 GHz. However, a dramatic increase in the phase noise close to the carrier on cooling, such as that recently reported by Mann *et al.* [6], did not appear, even when operating at levels well above the 1 dB gain compression point. The increase in slope on cooling seen in the noise data of [6] (from -10 dB/decade to ~ -30 dB/decade) might well be due to the importance of multiple traps in their devices, and the increase in their measured phase noise with increasing input power suggests that an up-conversion process is occurring. The up-conversion mechanism appears to be much less important for our devices. Our data suggest that it should be possible to build an oscillator with exceptionally good short-term frequency stability by using a cooled amplifier such as this with a high-Q resonator in a feedback loop, as shown in the next section. Also, it might be possible to improve the phase noise somewhat by careful choice of devices: we note that our room-temperature phase noise is $\sim$ 5 dB higher than that given by Walls and Felton [10] for a 10.6 GHz GaAs MESFET amplifier.

IV. Predicted Performance of a Feedback Oscillator Based on an 8.5 GHz Cooled GaAs FET Amplifier

The problem of phase noise in feedback oscillators has been considered by Leeson [11] and in more detail by Sauvage [12]. Internal phase fluctuations in the active element cause phase fluctuations at the oscillator output. If the oscillator output is taken from the output side of the amplifier, then the output phase noise spectrum, $S_{\varphi o}(f)$, is given by

$$S_{\varphi o}(f) = \left[1 + \left[\frac{\nu_0}{2Qf}\right]^2\right] S_{\varphi i}(f) \qquad (1)$$

where $S_{\varphi i}(f)$ is the spectral density of internal phase fluctuations (which corresponds to S_φ in Figs. 1 and 2), Q is the loaded Q of the resonator, ν_0 is the carrier frequency, and f is the sideband frequency. We see from Fig. 2 that below 2 K $S_{\varphi i}$ can be described approximately by $2\times10^{-10}/f$ Hz^{-1}. Using this value, we obtain for the spectral density of fractional frequency fluctuations, $S_y(f) = (f/\nu_0)^2 S_{\varphi o}(f)$:

$$S_y(f) = 2\times 10^{-10}\left[\frac{f}{\nu_0^2} + \frac{1}{4Q^2 f}\right]. \qquad (2)$$

The Allan variance, σ_y^2, is given approximately by [9]

$$\sigma_y^2(\tau) = \left(\frac{10^{-15}}{\tau}\right)^2 + \left(\frac{8.3\times 10^{-6}}{Q}\right)^2 \qquad (3)$$

for a measuring time τ, using an oscillator frequency of 8.5 GHz and a bandwidth of 1 kHz. We see from (3) that using a resonator with a loaded Q of 10^9 (easily possible with present-day superconducting technology), one would expect to obtain fractional frequency fluctuations $\sigma_y = \Delta f/f$ of $\sim 8.3\times 10^{-15}$ for times longer than 0.5 s. For shorter times the first term on the right-hand side of (3) becomes important. Since the oscillator would run at a power level of ~ 1 mW, thermal noise would not make a significant contribution to the Allan variance for measuring times longer than 0.5 s.

In comparison with this prediction, the frequency sources with the best short-term stability presently available commercially are quartz crystal oscillators, and the best of these exhibit fractional frequency fluctuations $\Delta f/f$ of $\sim 2\times 10^{-13}$ for 1 s $< \tau < 100$ s.

We are presently building a cooled loop oscillator using the 8.5 GHz amplifier in order to test our prediction.

V. Conclusions

We have measured the phase noise of an 8.5 GHz GaAs MESFET amplifier at temperatures from 1.7 K to 300 K. The observed flicker phase noise was found to vary only weakly with amplifier bias conditions and input power level, even when operating well past the 1 dB gain compression point. A cryogenic loop oscillator based on an amplifier with the level of intrinsic phase noise we observed at 2 K should have exceptionally good short-term frequency stability.

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Determination of the Scattering Matrix of Ring-Loaded Corrugated Waveguide Mode Converters

LUIZ C. DA SILVA

Abstract—**A previously developed method for the determination of the scattering matrix of cylindrical waveguide mode converters, which is based on the representation of the fields inside the corrugations by a small number of radial modes, is extended to mode converters with ring-loaded corrugations. The method, besides being accurate, reduces the computer time necessary for the computations.**

I. Introduction

Cylindrical waveguide mode converters, composed of a section of nonuniform corrugated waveguide, are employed as a matching device between smooth-walled input waveguides and corrugated horns, transforming the fundamental mode of the input waveguide into the desired mode of the horn [1], [2].

In a previous work [3] a method was developed for the determination of the scattering matrix of such converters. The method is based on the representation of the fields inside the corrugations by a small number of radial modes. In the present paper, this technique is extended to include converters with ring-loaded corrugations.

The main benefit resulting from the use of ring-loaded corrugations is an increase in the bandwidth of the converter.

A method of analysis for ring-loaded converters was previously developed by James and Thomas [2] on the basis of mode-matching techniques and the expansion of the fields inside the waveguide sections into modes propagating along the axis of the converter. The advantage of the present formulation is the saving of computer time it affords without loss of accuracy in the results.

II. Formulation of the Problem

The overall scattering matrix of the converter is obtained by dividing the structure into elementary sections, as shown in Fig. 1(a), calculating the scattering matrix of each section, and progressively cascading them. The scattering matrix of an elementary section is determined by following these steps:

(i) The elementary section is decomposed into three cascaded subsections, as shown in Fig. 1(b). Subsection I is the discontinuity between two smooth-walled waveguides, and subsection III is formed by a smooth-walled waveguide of length l. The scattering matrices of these subsections are calculated according to [1].

(ii) To obtain the scattering matrix of subsection II, it is initially transformed into the equivalent structure shown in Fig. 1(c), where metallic walls and magnetic surface current densities, $\pm \vec{M}_1$ and $\pm \vec{M}_2$, were placed at $\rho = b$ and $\rho = c$. In this way the

Manuscript received June 1, 1988, revised September 30, 1988 This work was supported in part by TELEBRÁS S A (Brazilian Telecommunications Agency) under Contract 168/86-JPqD

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IEEE Log Number 8825392

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